SPREADING OF NON-NEWTONIAN LIQUIDS UNDER THE GRAVITATIONAL FORCE

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A technique is proposed for determining the rheological constants based on the analysis of the spreading of tall cylindrical samples under the gravitational force.

The spreading of non-Newtonian liquids under the gravitational force is usually considered in the approximation of the theory of lubrication when the thickness of the liquid layer is much less than its diameter [1, 2]. In the present paper the analysis of spreading is given for high cylindrical samples $H_0/D_0 \sim 1$, where H_0 is the initial height of the sample, and D_0 is its initial diameter (Fig. 1). It is suggested that the rheology of a liquid is described by the relationship

$$\tau_{ij} = \left[\frac{\tau_0}{(I_2/2)^{1/2}} + K(I_2/2)^{(n-1)/2}\right]\dot{\gamma}_{ij}, \tag{1}$$

where τ_{ij} is the tensor of the shear stresses; τ_0 is the critical shear stress; $I_2 = 4\dot{\gamma}_{pq}\dot{\gamma}_{pq}$ is a second invariant of the tensor of velocities of deformation for an incompressible liquid; K and n are parameters of the medium characterizing its viscous properties; and $\dot{\gamma}_{ij}$ is the tensor of velocities of deformation.

The process of spreading of a liquid cylindrical sample on a horizontal surface is determined by the following dimensional parameters:

$$\rho,\ g,\ K,\ n,\ \tau_0,\ H_0,\ D_0,\ \alpha_0,\ t. \tag{2}$$

Here ρ is the liquid density; g is the acceleration of gravity; α_0 is the coefficient of the surface tension of the liquid; t is the time. In [3] it is shown that the ratio of the force of surface tension to the gravitational force is determined by the value of the dimensionless complex $\alpha_0 l/\rho g V_0$, where l is the perimeter of the surface layer of the liquid and V_0 is the volume of the sample. For samples with diameter $D_0 \gtrsim 1$ cm and not too high levels of spreading $H/H_0 \gtrsim 0.1$ (Fig. 1) an estimate gives that the gravitational force is much greater than the force of surface tension, and the value of the given dimensionless complex is small. Therefore, below we will ignore the effect of forces of surface tension on the process of spreading. In this approximation the parameter α_0 is not among the determining parameters.

We restrict ourselves to the case of slow flows when the equations of creeping flow are valid. Then the density does not have to be among the determining parameters itself [4], but only in the combination ρg. There remain seven dimensional parameters determining the process of spreading:

$$\rho g,\ K,\ n,\ \tau_0,\ H_0,\ D_0,\ t. \tag{3}$$

The dimensions of these parameters are as follows: $[\rho g]$, kg/(m·sec)^2; $[\tau_0]$, kg/(m·sec^2); [K], kg/(m·sec^{2-n}); $[H_0] = [D_0]$, m; [t], sec; n is a dimensionless parameter.

In order to determine corresponding dimensionless complexes, we construct the dimensional matrix [5], taking into account that two dimensionless complexes are evident right away: Π_1 = n and $\Pi_2 = H_0/D_0$:

	$[\rho g]$	$[K]$	$[\tau_0]$	$[H_0]$	$[t]$
kg	1	1	1	0	0
m	−2	−1	−1	1	0
sec	−2	$(n-2)$	−2	0	1

(4)

Elements of the dimensional matrix are exponents, in which the dimensional units enter into parameters (3). The rank of the dimensional matrix in the given case is equal to three, since the determinant of the third rank compiled, for example, from the third, fourth, and fifth columns is nonzero.

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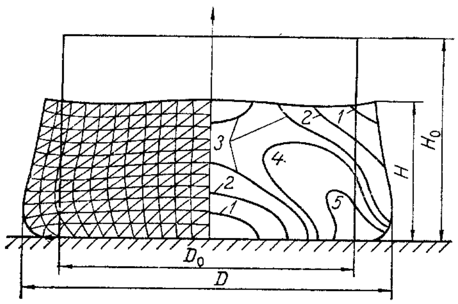

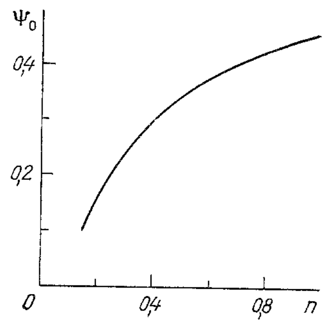

Fig. 1 Fig. 2

Fig. 1. Spreading of a block of a viscous liquid with K = 300 kPa·sec$^{0.46}$; n ≐ 0.46; D_0 = 0.02 m; H_0/D_0 = 0.5. Left-hand side, a finite-element network, right-hand side, distribution of the effective viscosity [1) 8 kPa·sec$^{0.46}$; 2) 5; 3) 4; 4) 3.6; 5) 3 kPa·sec$^{0.46}$].

Fig. 2. Dependence of Ψ_0 on the number n (H_0/D_0 = 0.7).

Any dimensionless complex characterizing the process can be written as

$$\Pi = (\rho g)^{q_1} K^{q_2} \tau_0^{q_3} H_0^{q_4} t^{q_5}. \tag{5}$$

Taking account of (4) we obtain the system of equations for determining the unknown exponents:

$$\begin{aligned} q_1 + q_2 &= 0; \\ -2q_1 - q_2 - q_3 + q_4 &= 0; \\ -2q_1 + (n-2)q_2 - 2q_3 + q_5 &= 0. \end{aligned} \tag{6}$$

By using (5) and (6) we obtain four independent dimensionless complexes which determine completely the process of spreading:

$$n, \quad \frac{H_0}{D_0}, \quad \frac{\tau_0}{\rho g H_0}, \quad t\left(\frac{\rho g H_0}{K}\right)^{1/n} = t_\delta, \tag{7}$$

where t_δ is a dimensionless time. All the remaining possible dimensionless complexes are functions of complexes (7). Any dimensionless quantity characterizing the process can be written as a function of complex (7). As such a characteristic we can take the dimensionless height of the spreading sample:

$$\frac{H}{H_0} = f_1\left[n, \frac{H_0}{D_0}, \frac{\tau_0}{\rho g H_0}, t\left(\frac{\rho g H_0}{K}\right)^{1/n}\right] \tag{8}$$

or the dimensionless ratio of the current height to the current diameter (see Fig. 1):

$$\frac{H}{D} = f_2\left[n, \frac{H_0}{D_0}, \frac{\tau_0}{\rho g H_0}, t\left(\frac{\rho g H_0}{K}\right)^{1/n}\right]. \tag{9}$$

Now we analyze Eqs. (8) and (9) in order to create a technique for determining spreading rheological constants. We consider some particular cases.

Newtonian Liquid (τ_0 = 0, n = 1). From (8) we obtain

$$\frac{H}{H_0} = f_1\left[\frac{H_0}{D_0}, t_\delta\right], \tag{10}$$

where $t_\delta = t\rho g H_0/\mu$; μ is the viscosity of the Newtonian liquid. We introduce the velocity of spreading v, defining it as the derivative of the instantaneous height with respect to time. Then from (10) we find

$$v \equiv \frac{dH}{dt} = \frac{\rho g H_0^2}{\mu} \Psi(H_0/D_0, t_\delta), \tag{11}$$

where

$$\Psi(H_0/D_0,\ t_\delta) = \frac{\partial f_1}{\partial t_\delta}.$$

For the maximal velocity of spreading (for t = 0) from (11) we obtain

$$v_{\max} = \frac{\rho g H_0^2 \Psi_0(H_0/D_0)}{\mu},$$

where $\Psi_0(H_0/D_0) = \Psi(H_0/D_0, 0)$ is the value of the function Ψ at the initial time. By knowing the numerical value of the function $\Psi_0(H_0/D_0)$ and measuring in the experiment the velocity of spreading v_{max}, we determine the viscosity of the liquid from the results of the experiments

$$\mu = \frac{\rho g H_0^2 \Psi_0(H_0/D_0)}{v_{\max}}. \tag{12}$$

The value of the function $\Psi_0(H_0/D_0)$ is not determined by the method of dimensional analysis and can be obtained only from a numerical solution of the problem.

Liquid with a Power Rheological Law ($\tau_0 = 0$). Equations (8) and (9) give

$$H/H_0 = f_1[n,\ H_0/D_0,\ t(\rho g H_0/K)^{1/n}],$$
$$H/D = f_2[n,\ H_0/D_0,\ t(\rho g H_0/K)^{1/n}]. \tag{13}$$

By differentiating the first of Eqs. (13) with respect to time, we obtain for the maximal spreading velocity

$$v_{\max} = \left(\frac{dH}{dt}\right)_{t=0} = \left(\frac{\rho g}{K}\right)^{1/n} H_0^{(n+1)/n} \Psi_0(n,\ H_0/D_0), \tag{14}$$

where

$$\Psi_0(n,\ H_0/D_0) = \left.\frac{\partial f_1}{\partial t_\delta}\right|_{t_\delta=0}.$$

From (14) we can express the coefficient of composition of the liquid:

$$K = \frac{\rho g H_0^{n+1} \Psi_0[n,\ H_0/D_0]}{v_{\max}^n}. \tag{15}$$

If the function $\Psi_0[n, H_0/D_0]$ is known, then with the help of (15) we can determine the coefficient of composition of the power liquid from the results of the spreading experiments. In this case it is necessary to know the second rheological parameter, the coefficient of nonlinearity of the liquid n. An expression for determining n can be obtained by using the methods of dimensional analysis and similarity theory.

We analyze the spreading of the two similar samples ($H_{01}/D_{01} = H_{02}/D_{02}$) from one liquid with different initial heights $H_{01} \neq H_{02}$. Then from (13) it follows that

$$(H/D)_1 = f_1[n,\ H_{01}/D_{01},\ t(\rho g H_{01}/K)^{1/n}],$$
$$(H/D)_2 = f_2[n,\ H_{02}/D_{02},\ t(\rho g H_{02}/K)^{1/n}].$$

If we compare these samples for the same level of spreading $(H/D)_1 = (H/D)_2$ then, due to similarity of the samples, the values of the third dimensionless complexes on the right-hand sides should be the same:

$$t_1(\rho g H_{01}/K)^{1/n} = t_2(\rho g H_{02}/K)^{1/n}. \tag{16}$$

Here t_1 and t_2 are the typical times of spreading of the first and second sample to the same level of spreading; this level can be specified arbitrary. From (16) we express the coefficient of nonlinearity of the liquid

$$n = \ln(H_{01}/H_{02})/\ln(t_2/t_1). \tag{17}$$

By measuring in the experiment the times t_1 and t_2 and knowing the initial heights of the similar samples, we can determine the coefficient of nonlinearity of the liquid. It is shown in [6] that by using the methods of similarity theory we can obtain analytic expressions for determining the relative viscosity of a non-Newtonian suspension from the results of spreading experiments. However, for determining the second rheological parameter K it is insufficient to use only methods of similarity theory and dimensional analysis. It is required to also use results of the numerical calculation of the process of spreading for high samples under the gravitational force [7].

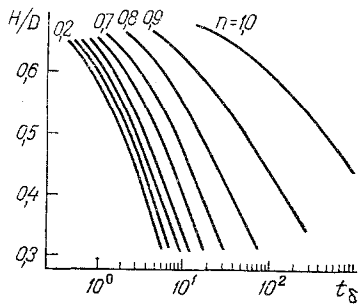

Fig. 3. Calculated dependences of the ratio H/D on the dimensionless time t_δ for different n (H_0/D_0 = 0.7).

Mathematically this problem falls into the class of problems for differential equations with the so-called "free boundaries" (in our case, free surface). The shape of this boundary is unknown and is the desired quantity. This condition is responsible for the nonlinearity of the problem since the unknown boundary or its part is determined from the unknown solution. The linearization is usually achieved by the introduction of an iteration procedure on determining the shape and the location of the boundary, starting from a certain known, initial position in the space. At each iteration or time interval it is required to solve the problem determining the components of the vector of velocity and pressure, i.e., the corresponding unknowns in the Navier-Stokes equations. If we consider the spreading of high-viscosity liquids, then we can ignore the convective derivative with respect to the velocity in the equations of motion. For solving a physically nonlinear problem, i.e., when the liquid under investigation is non-Newtonian, it is necessary to iterate on this nonlinearity at each time interval. After the velocity field has been determined, the front of the free surface is moved according to the velocity found and the kinematic conditions.

The above algorithm has been realized on the basis of the finite-element method with the use of the variational principle. The position of the cylinder is known initially. Its longitudinal cross section is covered by a network of finite triangular elements, the coordinates of which are known. The functional is minimized numerically and the nodal velocity components are determined. A certain time interval is specified, and new coordinates of the nodes are determined on this interval. Then the velocity field is determined again and so on, up to the desired values of the spreading time. At each time interval there is a possibility for determining different characteristics of lowering: pressure distribution, intensity of velocities of deformation, viscosity in the entire volume of the sample. In Fig. 1, a certain moment of the process of spreading, the shape of the finite element network, and the distribution of the effective viscosity are given.

In Fig. 2, results of the calculation of the dimensionless function $\Psi_0[n, H_0/D_0]$ are given for different values of the coefficient of nonlinearity of the liquid. All calculations were conducted for the ratio H_0/D_0 = 0.7. In that case the experiment provided for sufficient stability of the sample.

After determining the coefficient of nonlinearity n in the experiment, we can determine the second rheological parameter, the coefficient of composition K, from Eq. (16). In order to do this, it is necessary to measure the maximal velocity of spreading v_{max} in the experiment. The viscosity of the Newtonian liquid (n = 1) can be determined from Eq. (12).

The coefficient of composition can also be found in another way. We express formally the coefficient of composition from the definition of the dimensionless time (7):

$$K = \rho g H_0 (t/t_\delta)^n. \tag{18}$$

Here, the values of the ordinary time t and the dimensionless time t_δ are taken for one level of spreading (i.e., value of H/D). The value of t is determined from the experiment (we can make use of results of experiments for determining n), and t_δ is found from the calculated data. In Fig. 3, the results of numerical calculations are given for the dependence of the ratio H/D on the dimensionless time for different values of the coefficient of nonlinearity of the liquid (H_0/D_0 = 0.7). From the given nomogram we can readily determine the value of the dimensionless time for any $H/D < H_0/D_0$.

NOTATION

H_0, D_0, initial height and diameter of the sample; H, D, instantaneous height and diameter of the sample; τ_{ij}, $\dot{\gamma}_{ij}$, tensors of shear stresses and velocities of deformations; I_2, the second invariant of the tensor of velocities of deformations; τ_0, K, n, rheological constants; ρ, density; g, acceleration of gravity; α_0, coefficient of surface tension; t, t_δ, dimensional and dimensionless times; Π_i, dimensionless complexes; v, v_{max}, velocity of lowering, maximal velocity of lowering; Ψ, Ψ_0, dimensionless functions.

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EFFECTIVE VISCOPLASTICITY PARAMETERS OF SUSPENSIONS

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The effective field method is used to determine effective parameters of suspensions consisting of rigid ellipsoidal inclusions in a nonlinear viscoplastic matrix.

1. General Relationships. Within a macroregion z with characteristic function Z we will consider a suspension containing a statistically large number of rigid ellipsoidal inclusions and an incompressible viscoplastic matrix, the mechanical properties of which are described by a dissipative function

$$D = k\sqrt{\varepsilon_{ij}\varepsilon_{ij}} + \frac{1}{2}\eta(\varepsilon_{kl})(\varepsilon_{ij}\varepsilon_{ij}) + a e_{ij}\varepsilon_{ij}. \tag{1}$$

For definiteness, we will consider the variant of a power-law liquid $\eta(\varepsilon_{k\ell}) = \eta_0(I_2')^{(n-1)/2}$, where $I_2' = э_{k\ell}э_{k\ell}$ is the second invariant of the deformation rate deviator $э_{k\ell} = \varepsilon_{k\ell} - \varepsilon_{ii}/3$, $\varepsilon_{ii} = 0$ from the incompressibility condition.

The matrix contains a Poisson set $X = (V_k, x_k, a^i, \omega_k)$ of ellipsoidal rigid inclusions v_k with characteristic functions V_k, centers x_k forming a Poisson set, semiaxes a^i ($a^1 > a^2 > a^3$) and set of Euler angles ω_k with the inclusions having identical dimensions but various orientations. We will assume the random fields X, σ, ε, э ergodic and statistically homogeneous, so that averaging over the set can be replaced by averaging over characteristic volumes:

$$\langle (\cdot) \rangle_\alpha = \bar{v}_\alpha^{-1} \int (\cdot) V_\alpha(x)\, dx,\ \bar{v}_\alpha = \operatorname{mes} v_\alpha,\ \alpha = 0,\ 1,\ \ldots,$$
$$\langle (\cdot) \rangle = (\operatorname{mes} z)^{-1} \int (\cdot) Z(x)\, dx,$$

$V_0 = Z - V$, $V = \sum_k V_k$. In the future we will use the notation $\langle \cdot \mid x_1; x_2 \rangle$ for the conditional average over the set X, where at x_1, x_2 we have inclusions with $x_1 \neq x_2$.

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